Technical Comments

Comment on "Compressible Boundary-Layer Equations Solved by the Method of Parameter Differentiation"

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IN a recent Note by Narayana and Ramamoorthy,[1] the simultaneous nonlinear ordinary differential equations, resulting from a similarity analysis of the compressible boundary-layer equations, were solved by the method of parameter differentiation. Upon differentiating the equations with respect to β, the resulting equations [Eqs. (3) and (4)] were linearized, which were then separated into two systems of differential equations by using a two-term expansion of g and T [Eqs. (7) and (8)]. Due to the fact that T appears in Eq. (9), which contains an unknown parameter μ, the solution required iteration. Since the value of the method of parameter differentiation lies on the elimination of iteration, this simply destroys the purpose of using this method.

Iteration can in fact be eliminated by expanding g and T in a three-term form

$$g = g_1 + \lambda g_2 + \mu g_3 \tag{1}$$

$$T = T_1 + \lambda T_2 + \mu T_3 \tag{2}$$

which, upon substituting into Eqs. (3) and (4) in the Note, gives

$$g_{1\eta\eta\eta} + f g_{1\eta\eta} + f_{\eta\eta} g_1 + \beta(T_1 - 2f_\eta g_{1\eta}) + (S - f_\eta^2) = 0 \tag{3}$$

$$T_{1\eta\eta} + f T_{1\eta} + g_1 S_\eta = 0 \tag{4}$$

$$g_1(0) = g_{1\eta}(0) = g_{1\eta\eta}(0) = 0, \qquad T_1(0) = T_{1\eta}(0) = 0$$

$$g_{2\eta\eta\eta} + f g_{2\eta\eta} + f_{\eta\eta} g_2 + \beta(T_2 - 2f_\eta g_{2\eta}) = 0 \tag{5}$$

$$T_{2\eta\eta} + f T_{2\eta} + g_2 S_\eta = 0 \tag{6}$$

$$g_2(0) = g_{2\eta}(0) = g_{2\eta\eta}(0) = 0, \qquad T_2(0) = 0, \qquad T_{2\eta}(0) = 1$$

$$g_{3\eta\eta\eta} + f g_{3\eta\eta} + f_{\eta\eta} g_3 + \beta(T_3 - 2f_\eta g_{3\eta}) = 0 \tag{7}$$

$$T_{3\eta\eta} + f T_{3\eta} + g_3 S_\eta = 0 \tag{8}$$

$$g_3(0) = g_{3\eta}(0) = 0, \qquad g_{3\eta\eta}(0) = 1, \qquad T_3(0) = T_{3\eta}(0) = 0$$

Table 1 Initial slopes for negative β's

S_w	β	$f''(0)$ Present method	Ref. 1	Present method	Ref. 1
0.0	−0.30	0.3184	0.3178	0.4261	0.4261
	−0.25	0.3572		0.4384	
	−0.20	0.3875	0.3875	0.4475	0.4477
	−0.15	0.4125	0.4125	0.4546	0.4547
	−0.10	0.4340	0.4340	0.4605	0.4605
	−0.05	0.4528		0.4654	
	0.00	0.4696	0.4696	0.4696	0.4696
0.2	−0.14	0.3846	0.3841	0.3594	0.3590
	−0.12	0.3987		0.3623	
	−0.10	0.4120	0.4122	0.3649	0.3650
	−0.05	0.4424	0.4425	0.4708	0.3708
	0.00	0.4696	0.4696	0.3757	0.3757

The boundary conditions are specified such that

$$\mu = g_{\eta\eta}(0) \quad \text{and} \quad \lambda = T_\eta(0) \tag{9}$$

From the boundary condition at infinity, namely, $g_\eta(\infty) = T(\infty) = 0$, we then get two algebraic equations in λ and μ, the solutions of which are

$$\lambda = \frac{T_1(\infty) g_{3\eta}(\infty) - T_3(\infty) g_{1\eta}(\infty)}{T_3(\infty) g_{2\eta}(\infty) - T_2(\infty) g_{3\eta}(\infty)} \tag{10}$$

$$\mu = \frac{g_{1\eta}(\infty) T_2(\infty) - g_{2\eta}(\infty) T_1(\infty)}{g_{2\eta}(\infty) T_3(\infty) - g_{3\eta}(\infty) T_2(\infty)} \tag{11}$$

Starting from the solution for $\beta = 0$, solutions for other values of β can be calculated by integrating the three systems of initial value problems given by Eqs. (3–8), from which λ and μ can be calculated from Eqs. (10) and (11), followed by integrating Eqs. (5) and (6) in the Note.[1] By this way, iteration can be eliminated completely.

Numerical results by this noniterative method are compared with those by the iterative methods of the authors[1] and of Cohen and Roshotko.[2] The agreement is excellent, as seen from Tables 1 and 2.

Table 2 Initial slopes for positive β's

S_w	β	$f''(0)$ Present method	Narayana & Ramamoorthy	Cohen & Reshotko	$S'(0)$ Present method	Narayana & Ramamoorthy	Cohen & Reshotko
0.0	0.0	0.4696	0.4696	0.4696	0.4696	0.4696	0.4696
	0.5	0.5814	0.5812	0.5806	0.4946	0.4942	0.4948
	1.0	0.6492			0.5072		
	1.5	0.6992			0.5155		
	2.0	0.7393	0.7387	0.7381	0.5215	0.5206	0.5203
0.2	0.0	0.4696	0.4696	0.4696	0.3757	0.3757	0.3757
	0.5	0.6554	0.6546	0.6547	0.4039	0.4036	0.4030
	1.0	0.7760			0.4180		
	1.5	0.8700	0.8695	0.8689	0.4272	0.4267	0.4261
	2.0	0.9489	0.9483	0.9480	0.4339	0.4334	0.4331

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References

[1] Narayana, C. L. and Ramamoorthy, P., "Compressible Boundary-Layer Equations Solved by the Method of Parameter Differentiation," *AIAA Journal*, Vol. 10, No. 8, Aug. 1972, pp. 1085–1086.

[2] Cohen, C. B. and Reshotko, E., "Similar Solutions for the Compressible Laminar Boundary Layer with Heat Transfer and Pressure Gradient," TR-1293, 1956, NACA.

Reply by Authors to Tsung Y. Na

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WE are thankful for Na's comments on our paper.[1] It is true that the expansion of the functions g and T in a three-term form as is done by Na, simplifies the procedure of solving the linear equations. However, the statement of Na that the value of the method of parameter differentiation lies in the elimination of iteration needs to be qualified for the following reasons. First, Eqs. (3–8) of his comments need iterations for solving if one uses predictor-corrector methods as has been done in Ref. 1. Secondly, the integrations of $\partial f/\partial\beta = g$ and $\partial S/\partial\beta = T$ [Eqs. (5) and (6) of Ref. 1] themselves need iterations if one uses trapezoidal rule as has been done in Ref. 1. Integration depending on iterative methods gives more accurate values for f and S and their derivatives which are to be used as coefficients in the equations for the components of g and T. To get a comparable accuracy by noniterative methods, one has to decrease the step length in β.

Reference

[1] Narayana, C. L. and Ramamoorthy, P., "Compressible Boundary-Layer Equations Solved by the Method of Parametric Differentiation," *AIAA Journal*, Vol. 10, No. 8, Aug. 1972, pp. 1085–1086.

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Comment on "Rates of Change of Flutter Mach Number and Flutter Frequency"

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IN a recent technical note, Rao[1] published expressions for the rates of change of flutter Mach number and flutter frequency with respect to the structural design variables, and made reference to an earlier paper by Rudisill and Bhatia.[2] Rao has derived the expressions for the derivatives by separately considering the real and imaginary parts, and his procedure requires evaluation of the cofactors of the flutter determinant. It was shown in the paper by Rudisill and Bhatia that the two unknown derivatives which appear on differentiating the flutter equation can be determined by separating the real and imaginary parts of the differentiated equation, and their expressions require the eigenvectors only and not the cofactors. Therefore, the footnote in Rao's note referring to the paper by Rudisill and Bhatia should read "Their equation instead (not also) requires the eigenvectors of the flutter problem in order to compute $\partial V_f/\partial X_k$." Rao also states in the same footnote that they (Rudisill and Bhatia) have not used the expression to predict the flutter behavior at the perturbed design. In fact, Rudisill and Bhatia used the flutter velocity derivatives in their search scheme to compute the change in structural design variables necessary to obtain the desired flutter velocity. This is clearly stated in the text and illustrated in Fig. 3 of Ref. 2.

References

[1] Rao, S. S., "Rates of Change of Flutter Mach Number and Flutter Frequency," *AIAA Journal*, Vol. 10, No. 11, Nov. 1972, pp. 1525–1528.

[2] Rudisill, C. S. and Bhatia, K. G., "Optimization of Complex Structures to Satisfy Flutter Requirements," *AIAA Journal*, Vol. 9, No. 8, Aug. 1971, pp. 1487–1491.

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IT appears that Rudisill and Bhatia misunderstood the statements made by Rao in Ref. 1. The following points might clarify the matter.

1) In general, both eigenvalues and eigenvectors are required in order to compute the rates of change of eigenvectors of any real eigenvalue problem. Similarly, the expressions derived by Rudisill and Bhatia[2] for $\partial V_F/\partial x_k$ require both the eigenvalues and eigenvectors of the complex eigenvalue (flutter) problem. But the expression derived by Rao[1] requires only the eigenvalues of the flutter problem. The first statement made in the footnote of Ref. 1 has to be understood in this context.

2) In structural optimization, the rates of change of any behavior quantity can be used in two distinct ways. Firstly, the partial derivatives of the behavior quantity (like flutter speed) could be directly used in any first order optimization method to compute the necessary gradients. Secondly, the partial derivatives could be used in the analysis routine to predict approximately the behavior of the structure at any perturbed design. The second statement in the footnote of Ref. 1 has to be taken to mean that the derivatives $\partial V_F/\partial x_k$ have not been used in Ref. 2 for the latter purpose.

References

[1] Rao, S. S., "Rates of Change of Flutter Mach Number and Flutter Frequency," *AIAA Journal*, Vol. 10, No. 11, Nov. 1972, pp. 1526–1528.

[2] Rudisill, C. S. and Bhatia, K. G., "Optimization of Complex Structures to Satisfy Flutter Requirements," *AIAA Journal*, Vol. 9, No. 8, Aug. 1971, pp. 1487–1491.

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